

2. Length, Area, and Volume

In 1960, the scientific community decided to adopt a common system of measurement so communication among scientists would be easier. The system they agreed to use is called the International System of Units (SI). Scientists all over the world use SI. When you are in the laboratory, all of your measurements will be taken using SI.

On this and the following pages you will learn about SI. When you have finished you will be able to

1. identify the SI base unit of length,
2. identify the most commonly used prefixes, and
3. calculate length, area, and volume using the SI base unit for length and units derived from that base unit.

Length

SI is a decimal system. Our money system is a decimal system: 10^2 , or 100, cents to the dollar. This means that all relationships between units of measurement are based on powers of 10. There are several prefixes that can be combined with an SI unit. These prefixes change the unit to mean some other power of 10. For example, the SI base unit of length is the meter. A meter is equal to 39.4 inches in the English system which we use in the United States. When the prefix *kilo* is added to the base unit *meter*, the word *kilometer* is formed (a derived unit meaning 1000 meters). When the prefix *centi* is combined with the base unit *meter* the word *centimeter* is formed (a derived unit meaning 1/100 of a meter).

In Figure 1, you will see two rulers that compare the metric scale of measurement with the English scale. Figure 1a illustrates that exactly 1 inch is equal to 2.54 centimeters. Figure 1b illustrates that exactly 1 centimeter is equal to 0.394 inches.

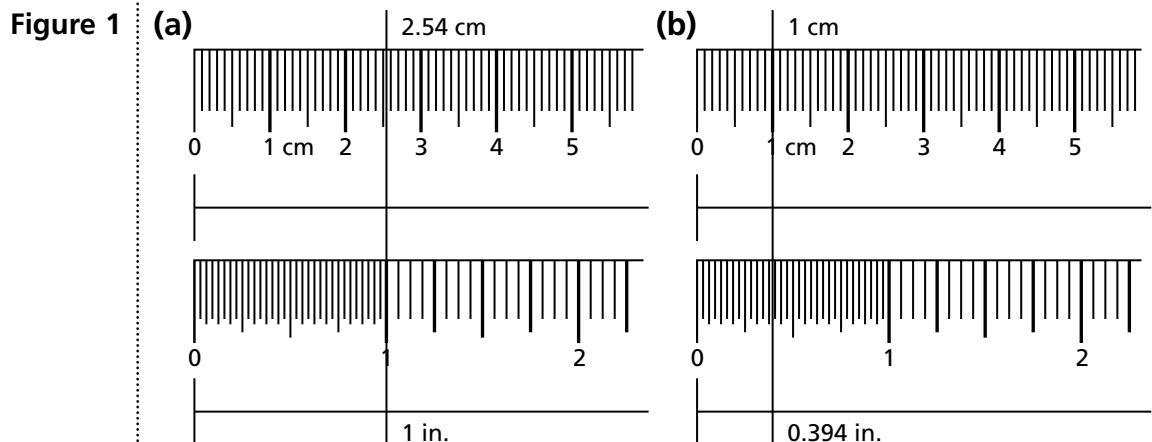


Table 1 lists some commonly used prefixes and their meanings. The prefixes modify the value of the base to describe very small lengths or very long distances, like the diameter of an animal cell or distances between cities. Cells are measured in micrometers, which is abbreviated μm . Distances between cities are measured in kilometers, which is abbreviated km. Find these prefixes in Table 1, and notice their relationship to the base unit.

Table 1 *Metric Prefixes*

Prefix	Symbol	Exponential	Rational	Meaning
tera	T	10^{12}	1,000,000,000,000	trillion
giga	G	10^9	1,000,000,000	billion
mega	M	10^6	1,000,000	million
kilo	k	10^3	1,000	thousand
hecto	h	10^2	100	hundred
deka	da	10^1	10	ten
deci	d	10^{-1}	1/10	one/tenth
centi	c	10^{-2}	1/100	one/hundredth
milli	m	10^{-3}	1/1,000	one/thousandth
micro	μ	10^{-6}	1/1,000,000	one/millionth
nano	n	10^{-9}	1/1,000,000,000	one/billionth
pico	p	10^{-12}	1/1,000,000,000,000	one/trillionth

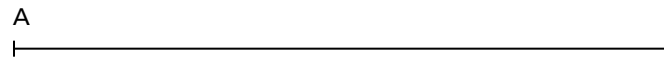
Because SI is based on powers of 10, we can easily convert from one derived unit to another. For example, there are 100 centimeters in a meter and there are 10 decimeters in a meter. Therefore, there are 10 centimeters in 1 decimeter. Find the answers to questions 1 through 6 using Table 1.

Practice Exercise 1

1. What prefix and unit represents 10 meters? _____
2. What derived unit represents 1/100 of a meter? _____
3. There are 1,000 millimeters in 1 meter (1 millimeter = 1/1,000 of a meter). Express 500 millimeters in meters. _____
4. The same measurement is often represented using different prefixes, depending on the purpose of the measurement. For example, the distance between Earth and the sun at its nearest position is usually written as 145 million kilometers. But the same measurement can be represented in meters to

emphasize the great distance. The sun is 145 billion meters from Earth at its nearest position in its annual orbit! The same distance can even be stated in terms of gigameters. How many gigameters is 145 million kilometers?

5. The length of line segment A can be represented in millimeters, centimeters, and meters. All of the measures are equivalent, and all represent the length of line segment A. Estimate the length of segment A, and then measure it. Express the measurement in each of the following units.



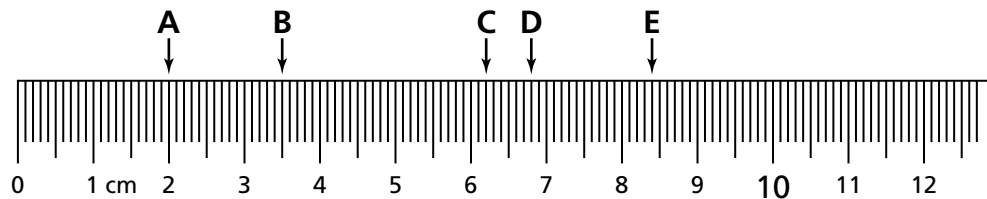
a. millimeters _____ b. centimeters _____ c. meters _____

6. Animal and plant cells are studied under a microscope. Some cells are so small that their parts are measured in micrometers. What part of a meter is a micrometer?
- _____

7. A millimeter is the smallest distance pictured on the metric ruler in Figure 2. Use the ruler to answer each of the following.

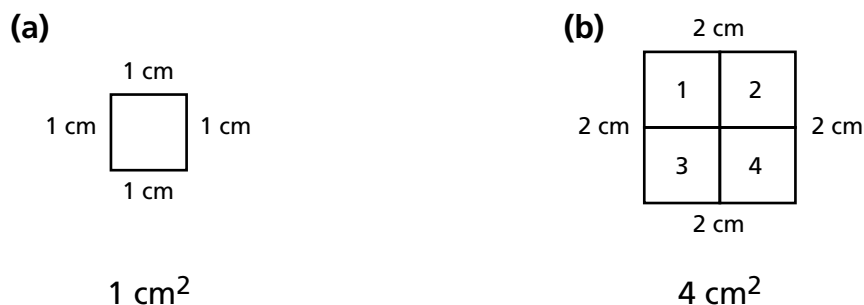
- a. A points to _____ mm or _____ cm
 b. B points to _____ mm or _____ cm
 c. C points to _____ mm or _____ cm
 d. D points to _____ cm or _____ mm
 e. E points to _____ cm or _____ mm

Figure 2



Area represents a measurement of two dimensions, length and width. The area of a region is the number of square units required to cover the region, without overlapping the squares, or allowing gaps. Figure 3a is a square measure 1 centimeter on a side that has an area of 1 square centimeter, denoted by 1 cm². Figure 3b is a square measure 2 centimeters on a side that has an area of 4 square centimeters.

Figure 3



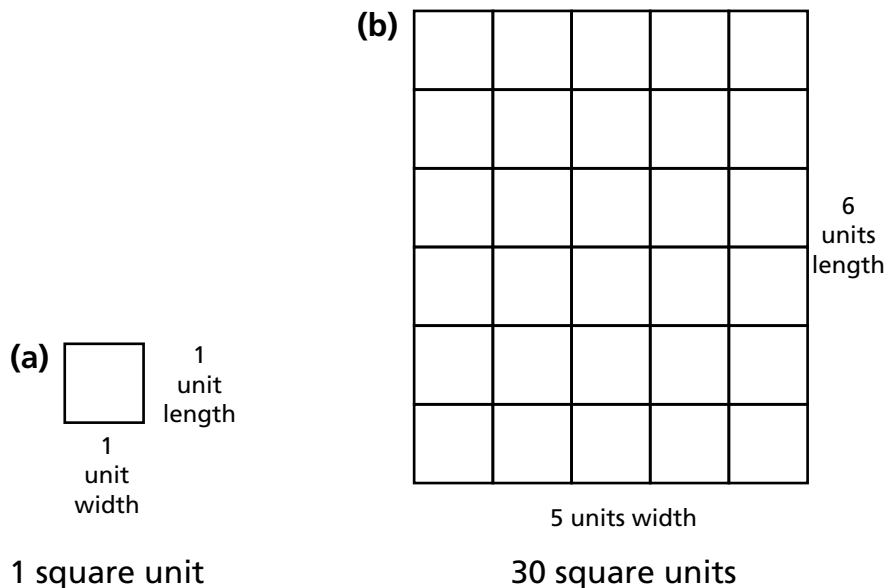
One way to measure area is to count the units contained in a given region. For example, suppose the square in Figure 4a is 1 square unit. The area of Figure 4b can be found by counting each of the same square units contained in Figure 4b. There are 30 non-overlapping squares and no gaps, so the area of Figure 4b is 30 square units.

To find the area of very large regions it may not be possible to count each unit. We can find the area of a region by counting the number of units on two adjacent sides. Figure 4b contains 5 rows of 6 squares each, or $5 \times 6 = 30$ squares. The area of any rectangle can be found by multiplying the length of any two adjacent sides (length and width).

$$\text{Area} = \text{length} \times \text{width} \qquad A = lw$$

If the length and width are given in centimeters, then the area is calculated in square centimeters (cm²). If the length and width are given in kilometers, then the area is calculated in square kilometers (km²). Both length and width must be stated in the same unit to calculate area.

Figure 4



Practice Exercise 2

1. A book measures 11 cm by 18 cm. How much area does the book occupy on the top of a desk? Draw a picture of the book. Label the length of each side and show your work.

2. Maria must replace a worn section of carpet. The damaged area measures 3 m long and 2 m wide. How much carpet does Maria need to buy?

3. A farmer wants to plant half of a field in corn and the other half in cotton. Each half of the field measures 50 m by 100 m. What is the total area the farmer wants to plant? _____

Remember that every measurement of length may be expressed using more than one prefix. Usually, the simplest expression is best. For example, it is better to state the diameter of a cell as 5 μm rather than 0.0000001 m, even though the two measurements are equivalent. The distance between cities is more appropriately stated in km rather than cm.

4. Choose the most appropriate metric unit for measuring each of the following:

a. Area of a sheet of notebook paper _____

b. Area of a desktop _____

c. Area of a classroom floor _____

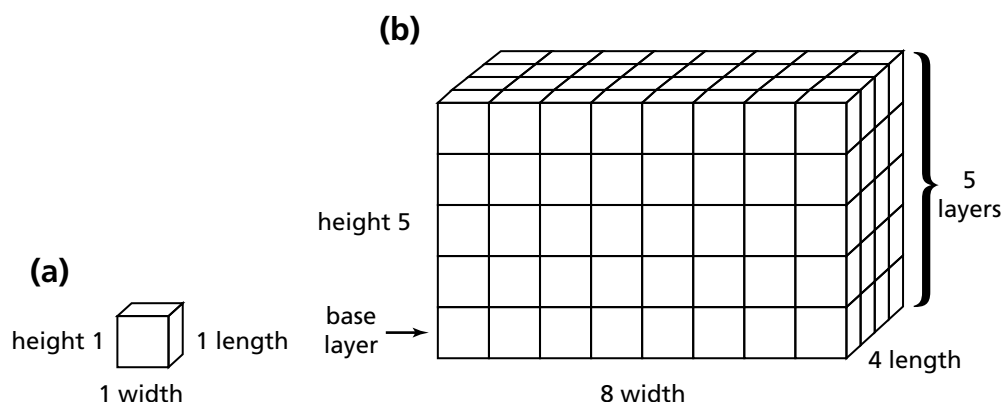
d. Area of an airport runway _____

5. A rectangular plot of land is to be seeded with grass. The plot to be seeded is 22 m by 28 m. If 1 kg of seed is needed for each 85 m² of land, how many kg of grass seed are needed? _____

Volume

As area is used to describe a two-dimensional region, volume represents a measure of a three-dimensional figure. Volume is how much space an object occupies. Volume can be shown with closely stacked cubes that fill a certain space, and have no gaps between them. Units of volume are based on cubes and are called cubic units. As shown in Figure 5, the volume of a rectangular solid can be measured by determining how many cubes cover the base and by counting how many layers of these cubes are needed to reach the height of the object.

Figure 5



As shown in Figure 5a, a unit cube measures 1 unit long, 1 unit wide, and 1 unit high, and equals 1 cubic unit. In Figure 5b, there are 8×4 , or 32, cubes in the base, and there are 5 such layers. Therefore, the volume of the object is $8 \times 4 \times 5$, or 160, cubic units. The volume of a rectangular solid can be found by multiplying the measures of length, width, and height.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height} \quad V = lwh$$

If the length, width, and height are given in centimeters, then the volume is calculated in cubic centimeters (cm^3). If the length, width and height are given in meters, then the volume is calculated in cubic meters (m^3). The length, width, and height must all be stated in the same unit before volume can be calculated. For example, to calculate the volume of a cube that is 2 dm long, 20 cm wide and 0.20 m high, each measurement must be converted to an appropriate common unit. Converting all of the units given, the length is 20 cm, the width is 20 cm and the height is 20 cm.

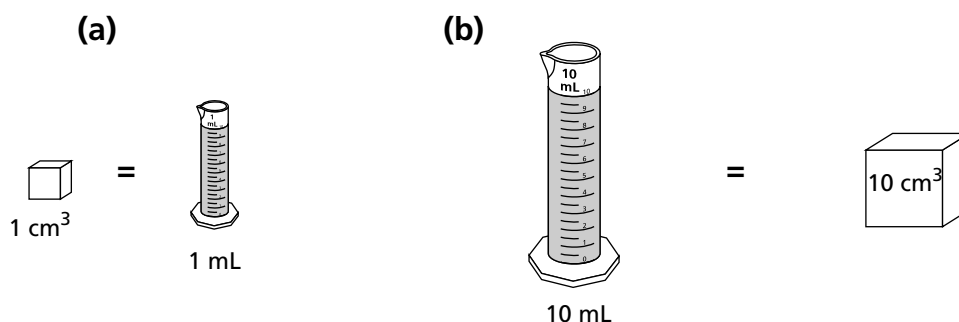
Practice Exercise 3

1. Calculate the volume of a cube that is 20 cm on a side. _____
2. What is the volume of a box 24 cm long, 12 cm wide, and 7 cm high?

3. Calculate the volume of a box which is 20 cm wide, 15 cm high and 78 cm long. _____

In the metric system, cubic units may be used for measuring the volume of space that a solid or liquid object occupies. Units such as liters and milliliters are usually used for capacity (the amount a container will hold) or liquid measures. By definition, 1 liter, symbolized by L, equals, or is the capacity of, 1 dm^3 . The liter is not an SI unit but is derived from other units. Because 1 L equals 1 dm^3 and 1 dm^3 equals 1000 cm^3 , then 1 cm^3 equals 1 mL ($1 \text{ cm}^3 = 1 \text{ mL}$).

Figure 6



4. A waterbed is 180 cm wide, 210 cm long, and 20 cm deep. How many liters of water can it hold? _____

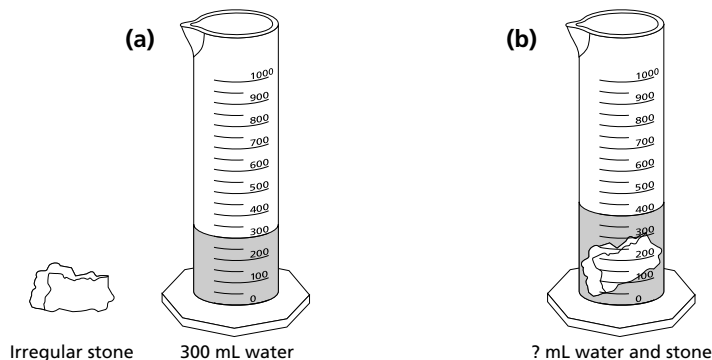
5. To understand the advantages of the metric system, try finding the volume of a waterbed in gallons using the following measurements: 6 ft long, 4.5 ft wide, and 9 in. deep. _____

A byte is not an SI unit, but you will see metric prefixes in many familiar phrases. The words formed may or may not be used to express a precise measurement, but the meaning of the prefix remains the same. See if you can answer the following questions.

9. How many bytes are in a gigabyte? _____
10. How many bucks are in a megabuck? _____

Activity

Figure 7



What is the Volume of the Stone?

Because we know that 1 mL is equal to 1 cm³ in volume, we are able to find the volume of solid objects that are irregular in shape and cannot be measured accurately. One way to do this is to submerge the object in a liquid such as water and to measure the volume of water displaced by the object.

Finding the Volume of an Irregular Solid

Supplies: One 500 mL graduated cylinder
1 stone about 3 cm in diameter
300 mL of water

Pour the 300 mL of water into the 500 mL graduated cylinder, and record 300 mL as your beginning volume of water. Place the stone into the graduated cylinder of water, and record the new level of water in the graduated cylinder.

Starting volume _____

Ending volume _____

Difference in volume _____

1. What is the difference between the first volume of water and the level of water in the beaker after the stone was added, in milliliters? _____

2. What is the difference in cubic centimeters? _____

3. What does your finding represent? _____

4. Explain how you know. _____

Review

1. Why did the world's scientific community decide to adopt a common system of measurement? _____

2. Explain what is meant by "decimal" system. _____

3. Name two other measurements that can be taken using the SI base unit for length and its derived units. _____
4. Name three units derived from the SI base unit for length, and explain their relationship to the base unit. _____

5. Convert each of the following.
 - a. 2.4 kilometers = _____ meters
 - b. 436 millimeters = _____ meters
6. Draw line segments that you estimate to be of the following lengths. Then check your estimates with a metric ruler.
 - a. 10 cm c. 1.5 dm
 - b. 29 mm d. 3 mm
7. Find the area of a rectangle 6 cm wide and 0.3 m long. _____

8. A rectangular piece of land is 1,250 m by 1,140 m. What is the area in square kilometers?
9. An Olympic pool is 50 m long and 25 m wide. If it is 2 m deep throughout, how many liters of water does it hold?

10. An unusual animal cell is being studied under a microscope. It is cubic in shape and is 6.5 μm on a side. What is the volume of the cell?
